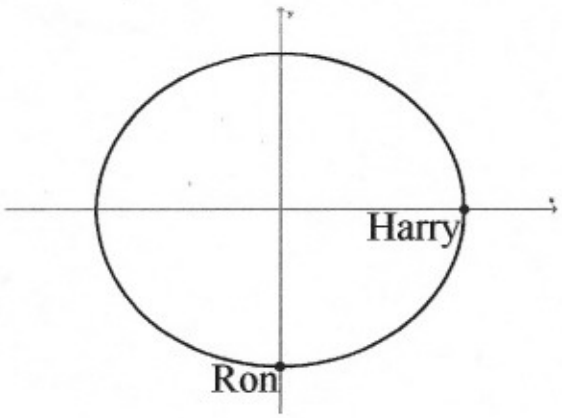
2. (10 points) Ron and Harry are both running counterclockwise on a circular track with radius 10 feet. Ron starts at the southernmost point and Harry is the easternmost point. Ron is running at 2 feet/sec and Harry completes one lap in 30 seconds.



(a) Give Harry's x and y coordinates after 3 seconds.

$\omega = \frac{1 \text{ rev}}{30 \text{ sec}} = \frac{2\pi \text{ rad}}{30 \text{ sec}} = \frac{\pi}{15}$ rad/sec

$\theta_0 = 0$ rad

$\theta = \omega t + \theta_0 = \frac{\pi}{15} \frac{\text{rad}}{\text{sec}} 3\text{sec} + 0 \text{ rad}$

$\theta = \frac{\pi}{5}$ rad $\quad r = 10$ ft

$x = r\cos(\theta) = 10\cos(\pi/5) \approx 8.090169944$

$y = r\sin(\theta) = 10\sin(\pi/5) \approx 5.877852523$

$\boxed{(x, y) \approx (8.09, 5.88)}$

(b) Give Ron's x and y coordinates after 50 seconds.

$v = 2$ ft/sec $\quad r = 10$ ft $\Rightarrow \quad \omega = \frac{v}{r} = \frac{2}{10} \frac{\text{rad}}{\text{sec}} = \frac{1}{5}$ rad/sec

$\theta_0 = -\frac{\pi}{2}$ rad

$\theta = \omega t + \theta_0 = \frac{1}{5} \frac{\text{rad}}{\text{sec}} 50 \text{ sec} - \frac{\pi}{2} \text{ rad} = 10 - \frac{\pi}{2}$ rad

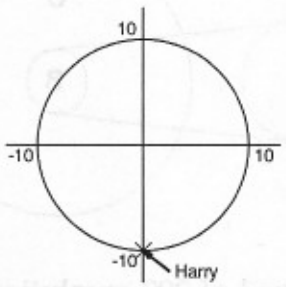
$\theta = 10 - \pi/2 \approx 8.429203673$ rad $\quad r = 10$ ft

$x = r\cos(\theta) = 10\cos(10 - \pi/2) \approx -5.440211109$

$y = r\sin(\theta) = 10\sin(10 - \pi/2) \approx 8.390715291$

$\boxed{(x, y) \approx (-5.44, 8.39)}$

5. (10 points) Harry is standing on the far southern outer edge of a merry-go-round of radius 10 feet. The merry-go-round is rotating counterclockwise with an angular speed of 15 revolutions per minute. Below we give a figure of this situation and we impose a coordinate system with the origin at the center of the merry-go-round.



Give the (x, y) coordinates of Harry after 2 seconds.

$x = r\cos(\theta)$

$y = r\sin(\theta)$

θ = angle in standard position

r = radius

$r = 10$ feet

$\omega = 15 \frac{\text{rev}}{\text{min}} \frac{2\pi \text{ rad}}{1 \text{ rev}} = 30\pi \frac{\text{rad}}{\text{min}}$

$t = 2 \text{ sec} \frac{1 \text{ min}}{60 \text{ sec}} = 0.0\overline{3} \text{ min} = \frac{1}{30} \text{ min}$

$\theta = \omega t$ = angle swept out from Harry's start location

$30\pi \frac{\text{rad}}{\text{min}} \; 0.0\overline{3} \text{ min} = \pi$ radians

Harry starts at $-\pi/2$ radians, so his angle in standard position is $\omega t + \theta_0 = \pi + -\pi/2 = \pi/2$ radians

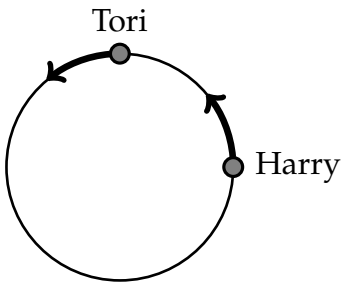
$x = 10\cos(\pi/2) = 0$

$y = 10\sin(\pi/2) = 10$

5. Tori and Harry are both running **counter-clockwise** around a circular track of radius 10 meters. Tori begins at the northernmost point and Harry begins at the easternmost point. Harry runs faster.

(a) **[4 points]** Tori first reaches the southernmost point after 8 seconds.

What is Tori's speed, in meters per second?



$\omega = \frac{\pi}{8}$ rad/sec

$v = \omega r = \frac{\pi}{8} \cdot 10 = \frac{5\pi}{4} \approx$ **3.93 m/s**

(b) **[6 points]** Harry begins running at the same time as Tori, and catches up to her in 11 seconds.

What is Harry's speed, in meters per second?

Tori has a head start of $\pi/2$ rad, so Harry runs $\frac{\pi}{2}$ rad more than her in 11 seconds. Tori runs $(\frac{\pi}{8})(11)$ radians, so Harry runs $(\frac{\pi}{8} \cdot 11) + \frac{\pi}{2} = \frac{15\pi}{8}$ radians in 11 seconds. His ω is $\frac{\frac{15\pi}{8}}{11} = \frac{15\pi}{88}$ rad/sec, and so:

$v = \omega r = \frac{15\pi}{88} \cdot 10 \approx$ **5.355 m/s**

(c) **[5 points]** Impose a coordinate system with units in meters and the origin at the center of the circle. After 80 seconds, what are Harry's coordinates?

$x = r\cos(\theta_0 + \omega t) + x_0$

$y = r\sin(\theta_0 + \omega t) + y_0$

$r = 10$, $t = 80$, $\theta_0 = 0$, $x_0 = 0$, $\omega = \frac{15\pi}{88}$, $y_0 = 0$

$x = 10\cos(\frac{15\pi}{88} \cdot 80) \approx$ **4.154**

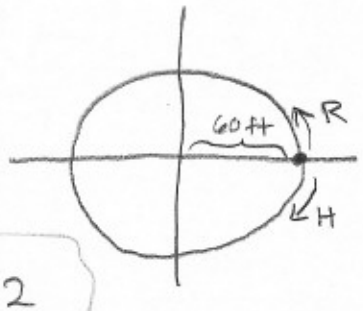
$y = 10\sin(\frac{15\pi}{88} \cdot 80) \approx$ **-9.096**

4. (10 points) Harry and Ron are both at the easternmost point of a circular track. The track has radius 60 feet. Ron runs in the counterclockwise direction at an angular speed of 0.02 radians per second. Harry runs in the clockwise direction.

(a) Impose a coordinate system with the middle of the track as the origin. Find the x and y coordinates of Ron after 30 seconds.

$x = r\cos(\omega t + \theta_0)$

$y = r\sin(\omega t + \theta_0)$



+5

$x = 60\cos(0.02(30) + 0) \approx 49.52$

$y = 60\sin(0.02(30) + 0) \approx 33.88$

(b) How fast (in feet per second) must Harry run in order to be at the same location as Ron after 30 seconds? (Hint: You may want to consider the angle that Harry will travel.)

$\theta = \omega t$ Ron travels $\theta = 0.02 \times 30 = 0.6$ radians

Harry travels $2\pi - 0.6 \approx 5.68318531$ radians (in the negative direction) } +3

For Harry $\rightarrow \omega = \dfrac{-5.68318531 \text{ radians}}{30 \text{ sec}} = 0.189439510239 \dfrac{\text{rad}}{\text{sec}}$ } +1

$v = \omega r = 0.189439510239 \times 60$

$= 11.3663706144$ feet/sec } +1

$v = 11.37$ ft/sec